The Local Wheeler-DeWitt Measure: A measure for the multiverse

Björn Friedrich (Heidelberg)

Based on 2205.09772 with Arthur Hebecker, Manfred Salmhofer, Jonah Strauss, Johannes Walcher

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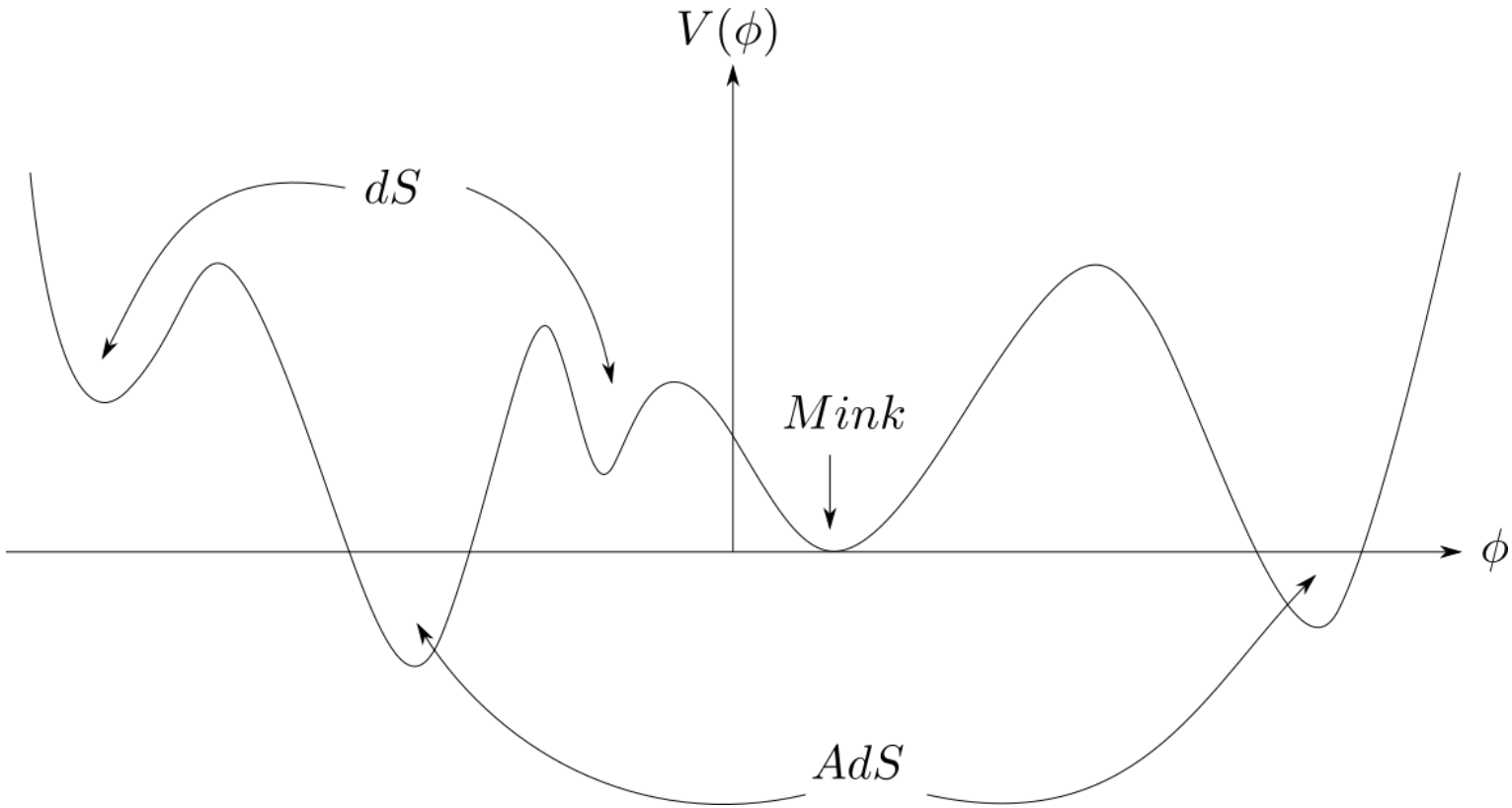
The landscape

- String theory prediction: Landscape of vacua
- Anti-de Sitter (AdS), Minkowski, de Sitter (dS)

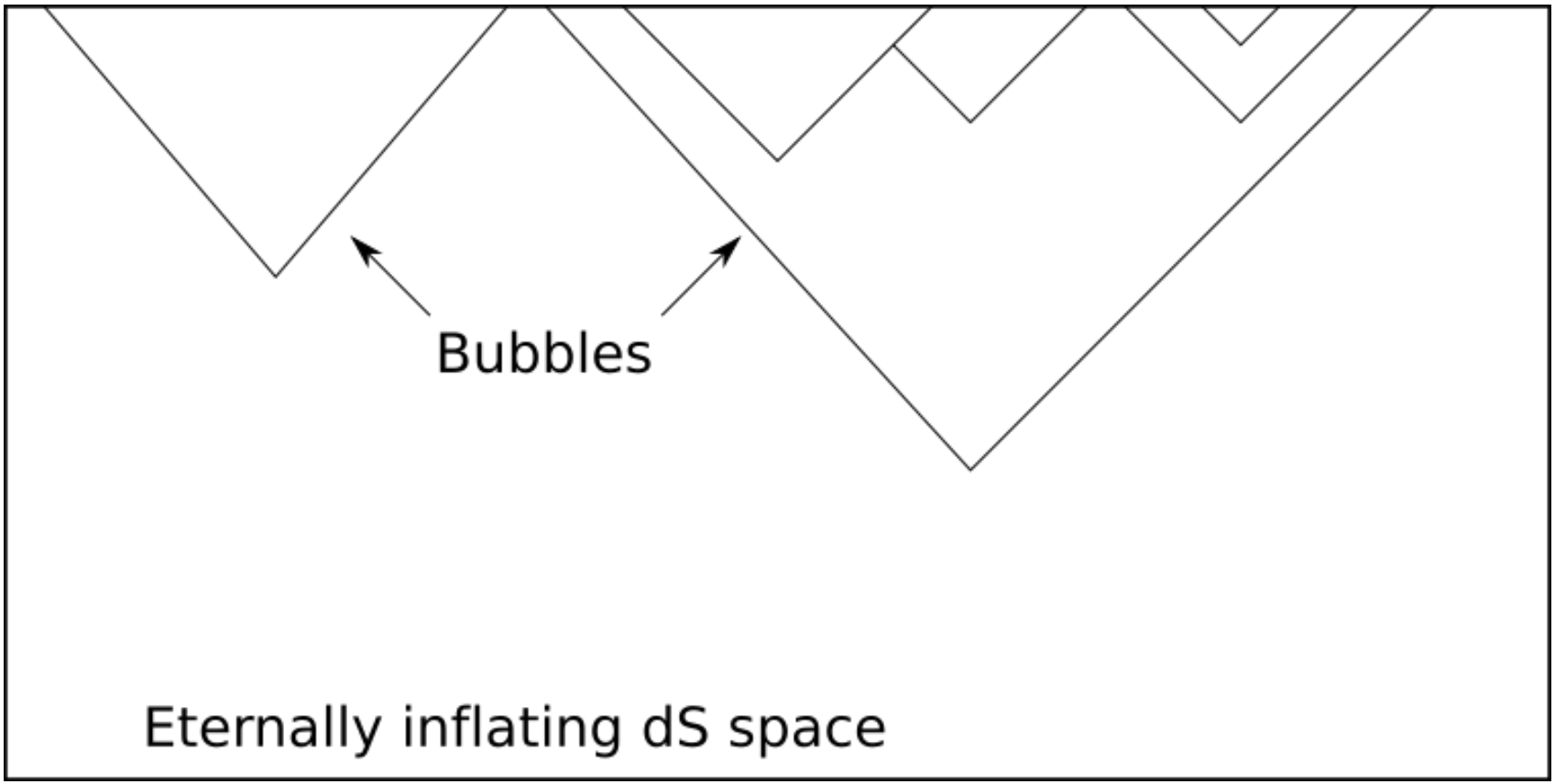


Caution:

The following is also relevant for landscapes without dS vacua and without eternal inflation.

Eternal inflation

- Eternal inflation: Inflation never ends
- Transitions between vacua: Nucleation of bubbles
- 'Everything that can happen, will happen infinitely many times.'



The measure problem (of eternal inflation)

Naive prediction for the outcome of a future measurement:

$$\frac{p_A}{p_B} = \frac{N_A}{N_B}$$

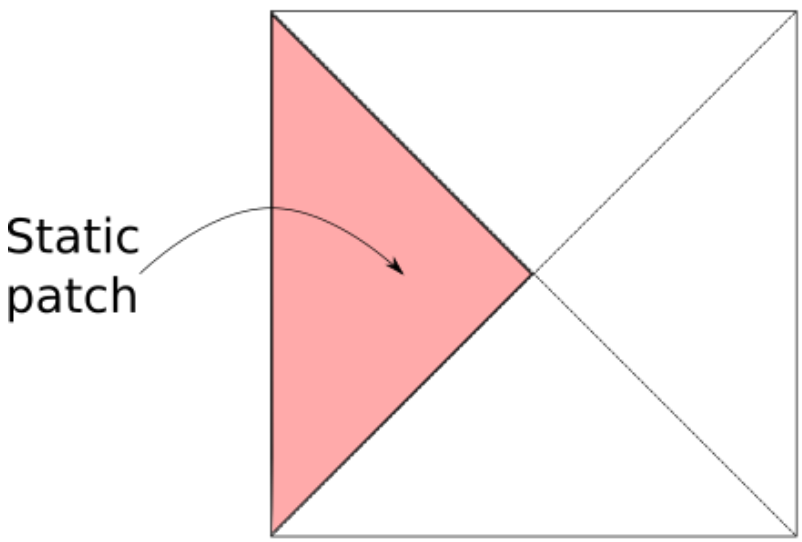
- p_A, p_B: Probabilities of measuring the result A or B
- N_A, N_B: Number of observers measuring result A or B
- 'Everything happens...': $N_A = \infty = N_B$

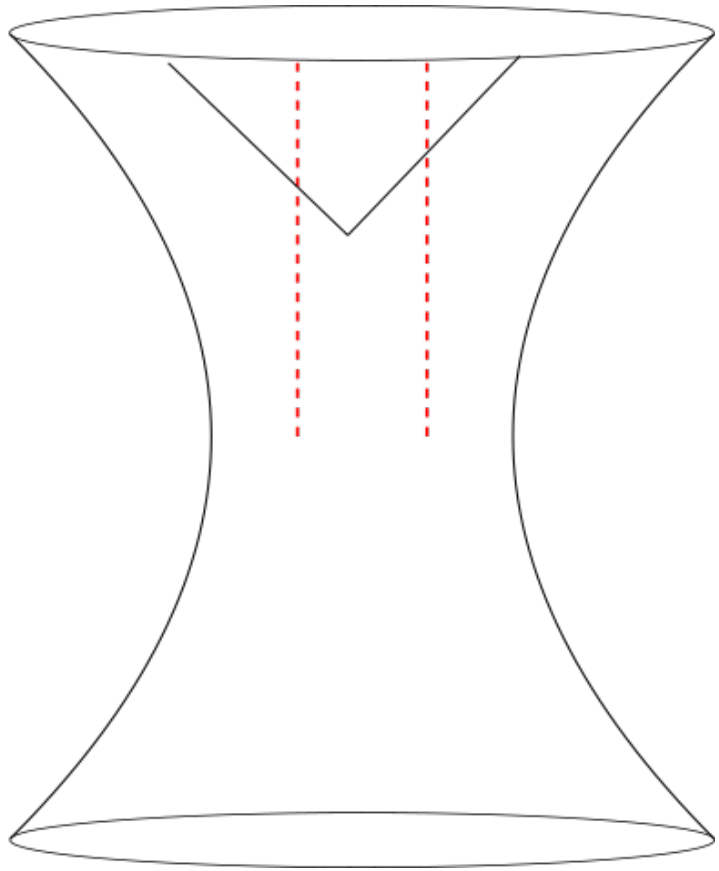
The measure problem: [Linde and Mezhlumian, 1993]

The problem of defining a probability measure for the set of possible outcomes of future measurements.

The local approach [Nomura, 2011, Garriga and Vilenkin, 2013]

- Focus on causally connected region of spacetime
- Size of expanding sphere is irrelevant





A model of the multiverse

Inputs:

- Quantum cosmology: $H\Psi = 0$ (WDW equation) [DeWitt, 1967]
- 'Cosmological Central Dogma': $\dim(\mathcal{H}_{dS}) = \exp(S_{dS}) < \infty$

  [Banks, 2001, Susskind, 2021]

Building a model:

- Hilbert space:

$$\mathcal{H} = \left(\bigoplus_{i \in dS} \mathcal{H}_i\right) \oplus \left(\bigoplus_{y \in Terminals} \mathcal{H}_y\right)$$

- For the dS-part of $\mathcal{H}$:

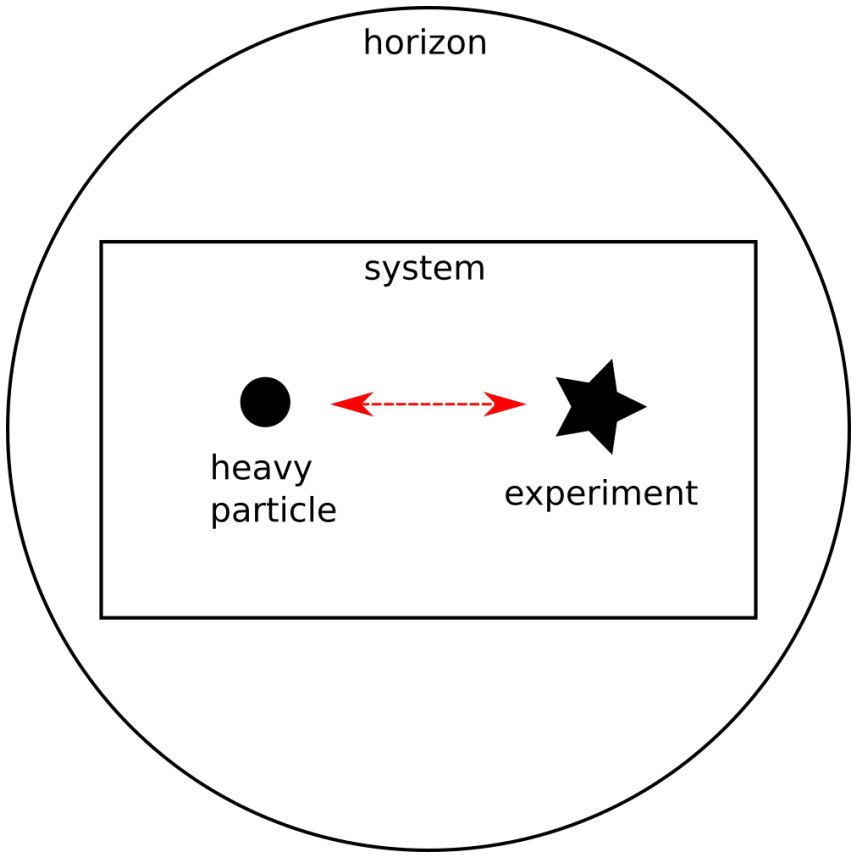
$$H = \begin{pmatrix} H_1 & \Delta_{21} & \cdots \\ \Delta_{21}^{\dagger} & H_2 & \\ \vdots & & \ddots \end{pmatrix}$$

- Ergodicity and random matrix theory: Consistent with semiclassical results

The problem of time

- $H\Psi = 0 \Rightarrow \Psi$ time-independent
- Emergence of time: Local correlation between observables

  [DeWitt, 1967, Lapchinsky and Rubakov, 1979, Banks, 1985]
- No need to refer to the scale factor variable

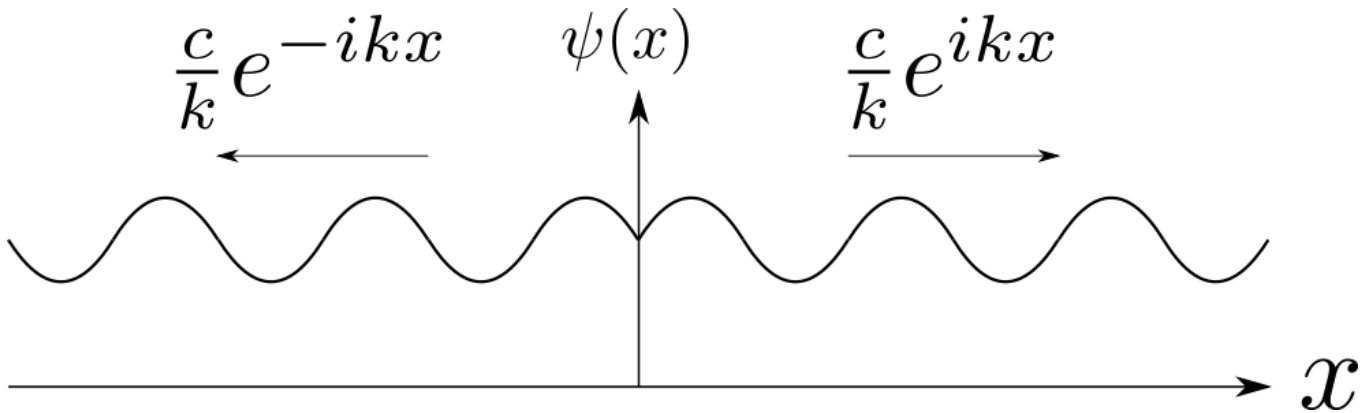


Terminal vacua and sources

- Tunneling from terminal to dS vacua allowed?
- Introduction of sources: Hartle-Hawking (no-boundary), Linde/Vilenkin (tunneling) [Hartle and Hawking, 1983, Linde, 1984, Vilenkin, 1984]

Example:

$$H\psi(x) = (\partial_x^2 + k^2)\psi(x) = 2ic\delta(x)$$



$$\nabla \cdot j(x) = 2\frac{|c|^2}{k}\,\delta(x)$$

Terminal vacua and sources

The case for the multiverse:

- $H\Psi = \chi\,,$ with χ a 'dS-like source'
- Boundary conditions: J 'runs out to infinity'

Connection to semiclassical results:

- Probability current: $J_i \propto \begin{cases} \exp(S_i) & \text{Hartle-Hawking} \\ \exp(-S_i) & \text{Linde/Vilenkin} \end{cases}$
- Equation for $p_i \equiv \| \Pr_i \Psi \|^2$

$$J_i = \sum_{j \in dS} (p_i \Gamma_{i\to j} - p_j \Gamma_{j\to i}) + p_i \sum_{y \in Terminals} \Gamma_{i\to y}$$

(A similar equation appears in [Garriga et al., 2006, Garriga and Vilenkin, 2013])

Making predictions

Project Ψ on subspace of interest and condition on observers

Anthropically viable vacua:

- Near-zero cosmological constant
- Inflation

Simplest prediction: Most likely anthropic vacuum

$$O_i \propto w_i \left(f_i J_i + \sum_{j \neq i} p_j f_{ji} \Gamma_{j \to i} \right)$$

- $w_j \in \{0, 1\}$: Classification of vacua as anthropically suitable or not
- f_i, f_{ij}: Fractions of creation/tunneling events ending in an inflationary state